# Introduction to Synchronous Machines

# **Definition:**

A synchronous machine is an ac rotating machine whose speed under steady state condition is proportional to the frequency of the current in its armature. The magnetic field created by the stator currents rotates at the synchronous speed ,and that created by the field current on the rotor is rotating at the synchronous speed also, and a steady torque results. So, these machines are called synchronous machines because they operate at constant speeds and constant frequencies under steady-Synchronous machines are commonly used as state conditions. generators especially for large power systems, such as turbine generators and hydroelectric generators in the grid power supply. Because the rotor speed is equal to the synchronous speed of stator magnetic field, synchronous motors can be used in situations where constant speed drive is required. Since the reactive power generated by a synchronous machine can be adjusted by controlling the magnitude of the rotor field current, unloaded synchronous machines are also often installed in power systems for power factor correction or for control of reactive kVA flow. Such machines, known as synchronous condensers, and may be more economical in the large sizes than static capacitors.

The bulk of electric power for everyday use is produced by polyphase synchronous generators( alternators), which are the largest single-unit electric machines in production. For instance, synchronous generators with power ratings of several hundred megavolt-amperes (MVA) are fairly common, and it is expected that machines of several thousand megavolt- amperes will be in use in the near future. Like most rotating machines, synchronous machines are capable of operating both as a motor and as a generator. They are used as motors in constant-speed drives, and where a variable-speed drive is required , a synchronous motor is used with an appropriate frequency changer such as an inverter. As generators, several synchronous machines often operate in parallel, as in a power station. While operating in parallel, the generators share the load with each other; at a given time one of the generators may not carry any load. In such a case, instead of shutting down the generator, it is allowed to "float" on the line as a synchronous motor on no-load.

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# **Construction of synchronous machines :**

The synchronous machine has 3 phase winding on the stator and a d.c. field winding on the rotor.

# 1. Stator :

It is the stationary part of the machine and is built up of sheet-steel laminations having slots on its inner periphery. A 3-phase winding is placed in these slots. The armature winding is always connected in star and the neutral is connected to ground.

# 2. Rotor :

The rotor carries a field winding which is supplied with direct current through two slip rings by a separate d.c. source. Rotor construction is of two types, namely;

(i) Salient (or projecting) pole type .

(ii) Non-salient (or cylindrical) pole type .

### (i) Salient pole type:

In this type, salient or projecting poles are mounted on a large circular steel frame which is fixed to the shaft of the alternator as shown in Fig. (1). The individual field pole windings are connected in series in such a way that when the field winding is energized by the d.c. exciter, adjacent poles have opposite polarities.

Low-speed alternators (120 - 400 r.p.m.) such as those driven by water turbines have salient pole type rotors due to the following reasons:

- (a) The salient field poles would cause .an excessive windage loss if driven at high speed and would tend to produce noise.
- (b) Salient-pole construction cannot be made strong enough to withstand the mechanical stresses to which they may be subjected at higher speeds.

Since a frequency of 50 Hz is required, we must use a large number of poles on the rotor of slow-speed alternators. Low-speed rotors always possess a large diameter to provide the necessary space for the poles. Consequently, salient-pole type rotors have large diameters and short axial lengths.



Fig. 1 salient pole rotor

### (ii) Non-salient pole(cylindrical) type :

In this type, the rotor is made of smooth solid forged-steel radial cylinder having a number of slots along the outer periphery. The field windings are embedded in these slots and are connected in series to the slip rings through which they are energized by the d.c. exciter. The regions forming the poles are usually left unslotted as shown in Fig. (2). It is clear that the poles formed are non-salient i.e., they do not project out from the rotor surface.



Fig. 2 cylindrical rotor

High-speed alternators (1500 or 3000 r.p.m.) are driven by steam turbines and use non-salient type rotors due to the following reasons:

- (a) This type of construction has mechanical robustness and gives noiseless operation at high speeds.
- (b) The flux distribution around the periphery is nearly a sine wave and hence a better e.m.f. waveform is obtained than in the case of salient-pole type.

Since steam turbines run at high speed and a frequency of 50 Hz is required, we need a small number of poles on the rotor of high-speed alternators (also called turboalternators). We can use not less than 2 poles and this fixes the highest possible speed. For a frequency of 50 Hz, it is 3000 r.p.m. The next lower speed is 1500 r.p.m. for a 4-pole machine. Consequently, turboalternators possess 2 or 4 poles and have small diameters and very long axial lengths.

### Classification of synchronous machines according to the form of excitation:

### 1. Brushes excitation systems:

The field structure is usually the rotating member of a synchronous machine and is supplied with a dc-excited winding to produce the magnetic flux. This dc excitation may be provided by a self-excited dc generator mounted on the same shaft as the rotor of the synchronous machine. This dc generator is known as *exciter*. The direct current generated inside exciter is fed to the synchronous machine field winding. In slow-speed machines with large ratings, such as hydroelectric generators, the exciter may not be self-excited. Instead, a *pilot exciter*, which may be self-excited or may have a permanent magnet, activates the main exciter.



# Fig. 1 Brushes Excitation system for a synchronous machine <u>2.Brushless systems</u>:

This type of excitation has a shaft-mounted bridge rectifier, that rotate with the rotor, thus avoiding the need for brushes and slip rings.



Fig.2 Brushless Excitation system for a synchronous machine

### 3.static systems:

This type of exctation is widely used in small size alternators, in which portion of the AC from each phase of synchronous generator is fed back to the field winding as a DC excitation through a system of transformer ,rectifiers and reactors. External source of a DC is necessary for initial excitation of the field windings.

### Advantages of stationary armature ;

The field winding of an alternator is placed on the rotor and is connected to d.c. supply through two slip rings. The 3-phase armature winding is placed on the stator.

This arrangement has the following advantages:

(i) It is easier to insulate stationary winding for high voltages , because they are not subjected to centrifugal forces .

(ii) The stationary 3-phase armature can be directly connected to load without going through large, unreliable slip rings and brushes.

(iii) Only two slip rings are required for d.c. supply to the field winding on the rotor. Since the exciting current is small, the slip rings and brush gear required are of light construction.

(iv) Due to simple and robust construction of the rotor, higher speed of rotating d.c. field is possible. This increases the output obtainable from a machine of given dimensions.

# <u>Cooling :</u>

Because synchronous machines are often built in extremely large sizes, they are designed to carry very large currents. A typical armature current density may be of the order of 10 A/mm<sup>2</sup> in a well-designed machine. Also, the magnetic loading of the core is such that it reaches saturation in many regions. The severe electric and magnetic loadings in a synchronous machine produce heat that must be appropriately dissipated. Thus the manner in which the active parts of a machine are cooled determines its overall physical structures. In addition to air, some of the coolants used in synchronous machines include water, hydrogen, and helium.

### **Damper Bars :**

So far we have mentioned only two electrical windings of a synchronous machine: the three-phase armature winding and the field winding. We also pointed out that, under steady state, the machine runs at a constant speed, that is, at synchronous speed. However, like other electric machines, a synchronous machine undergoes transients during starting and abnormal conditions. During transients, the rotor may undergo mechanical oscillations and its speed deviates from the synchronous speed, which is an undesirable phenomenon. To overcome this, an additional set of windings, resembling the cage of an induction motor, is mounted on the rotor. When the rotor speed is different from the synchronous speed, currents are induced in the damper windings .the damper winding acts like the cage rotor of an induction motor, producing a torque to restore the synchronous speed. Also, the damper bars provide a means of starting to the synchronous motors, which is otherwise not selfstarting. Fig6 shows the damper bars on a salient rotors



Fig .6 salient pole rotor showing the field winding and damper bars

# Differences between three phase induction machine and synchronous machine

Three phase induction machine	Synchronous machine
Stator phases either star or delta connected	Stator phase are star connected only
Rotor windings are not fed by electricity, currents flow through rotor due to induction process.	Rotor windings are fed by dc source.
Run below synchronous speed, as a motor Run above synchronous speed, as a generator	Run at synchronous speed for both motor and generator
Self starting, as a motor	Need damper bars to start, as a motor
Operate with lagging power factor only	Operate with lagging, leading, and unity power factor
Simple in construction, rugged, low maintenance, cheap, especially in squirrel cage type.	Complex in construction, expensive.
Not active in low speed operation, as a motor	Active in both low and high speed operation , as a motor
Not active in generating mode	Used widely in generation of electricity
Difficult in speed regulation, as a motor	Precise speed regulation, as a motor

# THREE PHASE STATOR WINDINGS

The stator windings for alternating-current motors and generators are alike. It should be noted that direct-current and alternating-current windings differ essentially by the former being of the closed-circuit type (through commutator) ,while alternating-current windings are of the open-circuit type.

### **Types of A-C Windings:**

With reference to the arrangements of coils used in three phase stator, windings may be divided into two general classes as follows:

# I. Distributed Windings:

- 1. Spiral or chain.
  - 2. Lap.
  - 3. Wave.

### **II. Concentrated Windings**:

1. Lap.

2. Wave.

### **Distributed Windings:**

An armature winding which has its conductors of any one phase under a single pole placed in several slots, is said to be **distributed**. When these conductors are bunched together in one slot per pole per phase, the winding is called **concentrated**. It is usual in a distributed winding to distribute the series conductors in any phase of the winding among two or more slots under each pole. A distributed winding has two principal advantages, first, a distributed winding generates a voltage wave that is nearly a sine curve, secondly, copper is evenly distributed on the armature surface, therefore, heating is more uniform and this type of winding is more easily cooled.

### Concentrated Windings:

The concentrated winding gives the largest possible emf from a given number of conductors in the winding. That is for a definite fixed speed and field strength in an alternator, the concentrated winding requires a less number of conductors than a distributed winding, but increases the number of turns per coil.

### Lap and Wave Windings:

Both distributed and concentrated windings make use of lap and wave connections. These arrangements are in principle the same as used in direct-current windings. The diagrams of Figs. 1 and 2 show distributed lap and wave windings.



Fig 2. Three-phase wave winding , using one slot per pole per phase, star-connected.

### **Chain Winding**:

In this winding as shown in Fig. 3 there is only one coil side in a slot. An odd or even number of conductors per slot may be used but several shapes of coils are required since the coils enclose each other. The number of coils required in this winding is also small compared with other windings. This type of winding is mainly used in alternating current generators.



Fig. 3 Three-phase chain winding

### Single layer and double layer winding :

In double layer winding two coil sides are located in each armature slot [Fig. 4]. If there is only one coil side located in each armature slot, the winding is called single layer.



Fig. 4 A double layer winding. One side of the coil lies in the top of the slot and the other side in the bottom of the of the slot.

### Full pitch and fractional pitch windings:

For a three-phase winding the total number of coil sides or the total number of slots should be just divisible by three (the number of phases) and sometimes by the number of poles. This will result in a **full pitch** winding, that is, a winding in which a coil spans exactly the distance between the centers of adjacent poles. If the coil spans less than this distance, so that its two sides are not exactly under the centers of adjacent poles at the same time, it is said to have a fractional-pitch. When a fractional pitch is used, the total number of slots per phase must be a whole number. The fractional-pitch coils are frequently used in a.c. machines for two main reasons. First, less copper is required per coil and secondly the waveform of the generated voltage is improved .The number of stator slots, divided by the number of poles gives a value of the pole pitch expressed in terms of the slots. Coil pitch is expressed as a fraction of the pole pitch, in slots, or in electrical degrees. In the case of a 6 pole machine having 72 stator slots, and a double-layer winding, the pole pitch would be 12 slots(72 / 6). If the coil pitch were given as 2/3, this would be  $120^{\circ}$  ( $2/3 \times 180^{\circ}$ ) or 8 slots (2/3 \* 12). A full coil pitch for this winding would be 180 degrees, or 12 slots. A full pitch winding is one in which the coil pitch is equal to the pole pitch, and a fractional pitch winding is one in which the coil pitch is not equal to the pole pitch. For the coils used in small machines round insulated wire is most employed. These coils are either wound in the slots by hand or assembled by use of specially formed coils wound in forms and insulated before being placed in the slots. Such formed coils are usually used except in cases where the slots are closed or nearly closed. For large machines where the amperes to be carried in each armature circuit is a large value, *copper straps* are frequently employed for making up the armature coils. In very large machines a *copper bar* is used instead of the copper straps. In such a case one bar serves as a conductor of a coil having one turn per slot. The copper bars are connected to the end connections of the coils by brazing, welding or bolting. In all cases, whatever the construction of the coil used, the slots must be properly insulated with mica, polyester film or other suitable insulating material according

to the adequate class of insulation. The **coil throw** refers to the start of coil from one stator slot to the finish of this coil at another stator slot  $\therefore$  Figures 5 (a), (b), (c), and (d) show four different coils, in each of them the pole pitch is of 12 slots while the coil pitch in (a) is 11 slots, in (b) is 13 slots, in (c) is 9 slots, and in (d) is 15 slots.



Fig. 5. Possible pitch for one coil per slot windings(pole pitch=24 slots/2poles=12) (a) coil throw 1 to 12, coil pitch=12-1=11 (b) coil throw 1 to 14, coil pitch=14-1=13 (c) coil throw 1 to 10, coil pitch=10-1=9, (d) coil throw 1 to 16, coil pitch=16-1=15.

### **Phase belt** and **phase spread**:

A group of adjacent slots belonging to one phase under one pole pair is known as **phase belt**. The angle subtended by a phase belt is known as **phase spread**. The 3-phase windings are always designed for  $60^{\circ}$  phase spread . Fig. (6) shows a 2-pole, 3-phase double-layer, full pitch, distributed winding for the stator of an alternator. There are 12 slots and each slot contains two coil sides. The coil sides that are placed in adjacent slots belong to the same phase such as(a1, a3) or (a2, a4) constitute a phase belt. Since the winding has doublelayer arrangement, one side of a coil, such as (a1), is placed at the bottom of a slot and the other side (- a1) is placed at the top of another slot spaced one pole pitch apart. Note that each coil has a span of a full pole pitch or 180 electrical degrees. Therefore, the winding is a full-pitch winding. Note that there are 12 total coils and each phase has four coils. The four coils in each phase are connected in series so that their voltages aid. The three phases then may be connected to form Y or  $\Delta$ -connection. Fig. (7) shows how the coils are connected to form a Y-connection. Any star diagram can be readily changed into a corresponding delta diagram by opening the star points and connecting the inner end of phase A to the outer end of phase B, the inner end of phase B to the outer end of phase C, and the inner end of phase C to the outer end of phase A.



Fig. 6

Fig. 7

### Simple rule for checking proper phase relationship in a three-phase winding:

A fundamental consideration when checking the instantaneous flow of current in a three phase circuit, is to imagine that when the current flows in the same direction in two legs of the circuit, it flows in the opposite direction in the third leg. This principle can be applied to both motors and generators. In Figure 8, it must be supposed that current flows in all three leads of the star connection toward the point of the star connection. And that in the case of a delta connection the current flows around the three sides of the delta in the same direction. Then in either case for a three-phase winding, the polarity of each of the pole-phase groups will alternate regularly around the winding and can be indicated by arrows as in Fig. 8. By the use of this scheme there is no chance for a reversal of a phase to be passed by not noticed when checking the winding.



**Fig. 8** Simple scheme of alternately reversing arrows of pole-phase groups to check correct phase polarity of a 3-phase winding. It is supposed in this case that current flows in the three leads toward the star points of the winding which are indicated thus (\*).

### Why the alternators have their windings connected in star?

Because the star connection has the following advantages over delta connection:

(i) To obtain a desired voltage between outside terminals of a generator, the voltage built up in any one phase need be only 58 percent $(1 / \sqrt{3})$  of the terminal value. Hence only 58 percent of the turns required for a  $\Delta$ -connected armature are necessary with a consequent lowering of insulation cost.

(ii) A star connected winding offers the advantage of a fourth or neutral lead making possible the advantages of a four-wire system, with or without grounded neutral.

(iii) The wave shape of a star-connected winding is improved, owing to the elimination of third harmonics and multiples of third harmonics from the terminal voltage. Fig.9 shows a star connected armature(stator), The terminal voltages  $E_{12}$ ,  $E_{23}$  and,  $E_{31}$  are 120 electrical degrees apart. In the third harmonics the e.m.f.'s of three phases (e01 ,e02 ,e03) are being in phase(3 X 120° =  $360^\circ$  = 0°), to obtain terminal voltages, the resultant is zero. Hence no third harmonics appears between terminals. The circulating currents from the third harmonics cause unnecessary losses and dangerous heating in  $\Delta$ -connected alternator, where it is not in the Y-connected alternator case. In addition the use of

a(5/6)th pitch winding in three-phase Y-connected generators reduces the fifth and seventh harmonics, if pesent, to almost nil, so the lowest harmonics that can be present is eleventh.



$$E_{12} = e_{02} + e_{10} = e_{02} - e_{01}$$
  

$$E_{23} = e_{03} + e_{20} = e_{03} - e_{02}$$
  

$$E_{30} = e_{01} + e_{30} = e_{01} - e_{03}$$

Fig. 9

## Windings factors:

The stator winding of synchronous machine is distributed over the entire stator. The distributed winding produces nearly a sine waveform and the heating is more uniform. Likewise, the coils of armature winding are not full-pitched i.e., the two sides of a coil are not at corresponding points under adjacent poles. The fractional pitched armature winding requires less copper per coil and at the same time waveform of output voltage is improved. The distribution and pitching of the coils affect the voltages induced in the coils. We shall discuss two winding factors:

(i) Distribution factor (Kd).

(ii) Pitch factor (Kp).

#### **Distribution factor** (Kd): **(i)**

A winding with only one slot per pole per phase is called a concentrated winding. In this type of winding, the e.m.f. generated/phase is equal to the arithmetic sum of the individual coil e.m.f.s in that phase. However, if the coils/phase are distributed over several slots in space (distributed winding), the e.m.f.s in the coils are not in phase (i.e., phase difference is not zero) but are displaced from each by the **slot angle**  $\beta$  (The angular displacement in electrical agrees between the adjacent slots is called slot angle). The e.m.f./phase will be the phasor sum of coil e.m.f.s. The distribution factor Kd is defined as:

$$K_{d} = \frac{\text{e.m.f. with distribute d winding}}{\text{e.m.f. with concentrated winding}}$$
$$= \frac{\text{phasor sum of coil e.m.f.s/phase}}{\text{arithmetic sum of coil e.m.f.s/phase}}$$

Note that numerator is less than denominator so that  $K_d < 1$ . Expression for K<sub>d</sub> = slot angle =  $\frac{180^{\circ} \text{ electrical}}{\text{No. of slots/pole}}$ 

Let

= slots per pole per phase

The distribution factor can be determined by constructing a phasor diagram for the coil e.m.f.s. Let m = 3. The three coil e.m.f.s are shown as phasors AB, BC and CD [See Fig. 10 (i)] each of which is a chord of circle with centre at O and subtends an angle  $\beta$  at O. The phasor sum of the coil e.m.f.s subtends an angle

 $m\beta$  (Here m = 3) at O. Draw perpendicular bisectors of each chord such as Ox, Oy etc [See Fig. 10 (ii)].

$$K_{d} = \frac{AD}{n \times AB} = \frac{2 \times Ax}{n \times (2Ay)} = \frac{Ax}{n \times Ay}$$
$$= \frac{OA \times \sin(n\alpha/2)}{n \times OA \times \sin(\alpha/2)}$$
$$K_{d} = \frac{\sin(n\alpha/2)}{n \sin(\alpha/2)}$$

Note that (  $m \beta$  ) is the phase spread.



Fig. 10

### (ii) Pitch factor (K<sub>p</sub>) :

....

A coil whose sides are separated by one pole pitch (i.e., coil span is 180° electrical) is called a full-pitch coil. With a full-pitch coil, the e.m.f.s induced in the two coil sides a in phase with each other and the resultant e.m.f. is the arithmetic sum of individual e.m.fs. However the waveform of the resultant e.m.f. can be improved by making the coil pitch less than a pole pitch. Such a coil is called short-pitch coil. This practice is only possible with double-layer type of winding The e.m.f. induced in a short-pitch coil is less than that of a fullpitch coil. The factor by which e.m.f. per coil is reduced is called pitch factor K<sub>p</sub>. It is defined as:

$$K_p = \frac{\text{e.m.f. induced in short - pitch coil}}{\text{e.m.f. induced in full - pitch coil}}$$

Consider a coil AB which is short-pitch by an angle  $\Theta$  electrical degrees as shown in Fig. (11). The e.m.f.s generated in the coil sides A and B differ in phase by an angle  $\Theta$  and can be represented by phasors  $E_A$  and  $E_B$  respectively as shown in Fig. (12). The diagonal of the parallelogram represents the resultant e.m.f.  $E_R$  of the coil.



Fig. 11

Fig. 12

Since  $E_A = E_B$ ,  $E_R = 2E_a \cos /2$ Pitch factor,  $K_p = \frac{e.m.f. \text{ in short - pitch coil}}{e.m.f. \text{ in full - pitch coil}} = \frac{2E_A \cos /2}{2E_A} = \cos /2$  $\therefore \quad K_p = \cos /2$ 

For a full-pitch winding,  $K_p = 1$ . However, for a short-pitch winding,  $K_p < 1$ . Note that  $\Theta$  is always an integer multiple of the slot angle  $\beta$ .

# SYNCHRONOUS GENERATOR

The machine which produces 3-phase electrical power from mechanical power is called an alternator or synchronous generator. Alternators are the primary source of all the electrical energy we consume. These machines are the largest energy converters found in the world.

#### <u>Alternator Operation :</u>

Like the dc generator, a synchronous generator functions on the basis of **Faraday's law, which state if the flux linking the coil changes in time, a voltage is induced in a coil. Stated in another form, a voltage is induced in a conductor if it cuts magnetic flux lines.** The rotor winding is energized from the d.c. exciter and alternate N and S poles are developed on the rotor. When the rotor is rotated by a prime mover, the stator or armature conductors are cut by the magnetic flux of rotor poles. Consequently, e.m.f. is induced in the armature conductors due to electromagnetic induction. The induced e.m.f. is alternating since N and S poles of rotor alternately pass the armature conductors. The frequency of induced e.m.f. is given by;

$$f = \frac{NP}{120}$$

Where N = speed of rotor in r.p.m., P = number of rotor poles.

The magnitude of the voltage induced in each phase depends upon the rotor flux, the number and position of the conductors in the phase and the speed of the rotor.

[Fig. 1 (i)] shows star-connected armature winding and d.c. field winding. When the rotor is rotated, a 3-phase voltage is induced in the armature winding. The magnitude of e.m.f. in each phase of the armature winding is the same. However, they differ in phase by 120° electrical as shown in the phasor diagram [Fig. 1 (ii)].



Fig. 1

Considering the round rotor synchronous generator in Fig.2, which shows a simple consentrated stator winding (1 slot / pole / phase).





and assuming that the flux density in the air gap is uniform implies that sinusoidally varying voltages will be induced in the three coils aa', bb' and cc' if the rotor, rotates at a synchronous speed,  $N_s$ . Also if  $\Phi_p$  is the flux per pole,  $\omega$  is the angular frequency, and T is the number of turns in phase **a** (coil aa'), then the voltage induced in phase **a** is given as:

 $e_a = \omega T \Phi_p \sin \omega t$ 

 $= \mathbf{E}_{\mathbf{m}} \sin \omega \mathbf{t}$ 

Where  $\mathbf{E}_{\mathbf{m}} = \boldsymbol{\omega} \mathbf{T} \boldsymbol{\Phi}_{p}$ , and  $\boldsymbol{\omega} = 2 \pi f$ .

Because phases **b** and **c** are displaced from phase **a** by  $+_120^\circ$ , the corresponding voltages may be written as

 $\mathbf{e}_{\mathrm{b}} = \mathbf{E}_{\mathrm{m}} \sin\left(\omega t - 120^{\mathrm{o}}\right)$ 

 $e_c = E_m \sin(\omega t + 120^\circ)$ 

These voltages are sketched in Fig. 3 and correspond to the voltages from a three-phase generator.



Fig. 3 A three-phase voltage produced by a three-phase synchronous generator

### Generated E.M.F. Equations:

The magnetic flux cut by one armature conductor, when the rotor of an alternator is made to revolve through one revolution, is  $\Phi_p * P$ , where  $\Phi_p$  is the magnetic flux per pole and P is the number of poles. If N is the speed in revolution per minute, then the flux cut per second becomes

### $\Phi_p * P * (N/60)$

Since 1 volt is generated when 1 weber of flux is cut per second, the average voltage generated in this conductor becomes

### $E_{av} = \Phi_p * P * (N/60) \text{ volts}$

If the total number of conductors on the armature is Z and they are connected into A parallel paths, the average voltage between terminals becomes

### $E_{av}$ per conductor \* (Z/A)

In case of alternating current generators the e.m.f. depends not only upon the total flux cut per second but also upon the way in which the flux and the conductors are distributed. A change in distribution of flux changes the relative values of the *maximum and effective (r.m.s.) e.m f.'s.* 

In addition, the e.m.f. built up in any one conductor, when considered vectorially, cannot always be added directly to that of another as there may be a phase displacement between them. The instantaneous values, though can be added algebraically, but in adding the effective values it is necessary to consider the phase differences between the different e.m.f.'s to be added. In order to take these factors in account, the flux distribution and winding types must be known.

Fig.4(a) shows the space distribution of the airgap flux of an alternator. In this case, the flux density B is assumed to be sinusoidal in space when measured around the inside periphery of the stator. This flux density B can be expressed as

### $B=B_{max} \sin\theta.$

where  $\boldsymbol{\theta}$  is measured from the position midway between the poles;  $\boldsymbol{B}$  is the flux density measured in webers per length of the field pole ( $\boldsymbol{L}$ ) as shown in Fig.4(b) per length of pole pitch arc; and Bmax is the maximum flux density produced by a pole.

The total flux per pole is

 $\Phi_p = \int_0^{\pi} LB_{max} \sin \theta \, d\theta = -|LB_{max} \cos \theta|_0^{\pi} = 2 LB_{max} \quad \text{Weber}$ ...(i)

As this flux wave is moved around the air gap the conductors **a** and **b** of the stator coil will have voltages induced in them.



Fig 4 Shapes of flux density waves

The voltage induced in conductor **a** will be

### $e_a = BL v$

...(ii) where v is the velocity of the flux wave in radians per second, or  $v = 2 \pi f$ , f being the frequency of the flux wave or e.m.f. induced.

The voltage in conductor **a** is

### $e_a = B_{max}L \ 2\pi f \sin \theta$

But  $\theta = \omega t$  where  $\omega$  is the angular velocity of the rotor in radians per second and t is the time of rotation from the position shown in Fig.4(a). Therefore,

$$e_a = B_{max}L \ 2\pi f \sin \omega t \qquad \dots (iii)$$

Substituting equation (i) in equation (iii)

$$e_a = (\Phi_p/2) * 2\pi f \sin \omega t$$
 ...(iv)

Likewise, the voltage induced in conductor **b** 

$$e_b = (\Phi_p/2) * 2\pi f \sin \omega t$$
 ...(v)

and the voltage at the terminals of the turn made up of the conductors **a** and **b** is

$$e_{turn} = \Phi_p 2\pi f \sin \omega t$$
 ...(vi)

If the single turn on the armature (stator) were replaced by a coil of T turns, the voltage per coil would be

$$e_{eoil} = \Phi_p 2\pi f T \sin \omega t$$
 ...(vii)

The effective value of generated e.m.f.

 $\mathbf{E}_{r.m.s} = \mathbf{E}_{max} / \sqrt{2} = (2\pi / \sqrt{2}) f \Phi_p T \qquad \text{volts (r.m.s)} \qquad \dots (\text{viii})$ 

 $E_{r.m.s} = 4.44 f \Phi_p T$  volts (r.m.s) ...(ix)

As the r.m.s. value is related to the average value so,

Form factor  $(k_f) = (r.m.s. value) / (average value)$ 

\* For sinusoidal wave of e.m.f.,  $k_f = 1.11$ 

 $\mathbf{E}_{\mathbf{r.m.s.}} = 4 \ k_f T f \Phi_p$  volts

Also, since the induced e.m.f.'s in the conductors of an alternator are not all in phase, the above relation for  $\mathbf{E}_{r.m.s.}$  must be multiplied by  $k_p$  and  $k_d$ , the pitch factor and the distribution (breadth) factor.

Therefore,

 $\mathbf{E}_{\mathbf{r.m.s.}} = 4 k_f k_p k_d f T \Phi_p \quad \text{volts per phase} \qquad \dots (\mathbf{x})$ 

\* For full pitched and concentrated windings  $k_d = 1$  and  $k_p = 1$ .

If the alternator is star connected, and neglecting the effect of armature reaction, the alternator line voltage is,

 $\mathbf{E}_{\mathbf{L}} = \sqrt{3} (\mathbf{E}_{r.m.s.} \text{ per phase})$ , so

 $\mathbf{E}_{\mathbf{L}} = \sqrt{3} (4 k_f k_p k_d f T \Phi_{p}) \quad \text{volts.}$ 

# Synchronous Generator Characteristics :

### (i) Magnetisation curve:

A plot of the exciting current versus terminal voltage of alternator is known as the magnetisation curve. This magnetisation curve is obtained by passing different values of currents in exciting windings, thereby giving, correspondingly different values of terminal voltage. The **no load magnetisation curve** is shown in Fig.5-I], which has the same general shape as *B-H* curve of armature steel.

The **full load magnetisation characteristics** with unity power factor and with 0.8 lagging power factor have been shown at (II) and (III) in Fig.5.



Fig. 5 Magnetization curves for alternator at different loading conditions

## (ii) Load characteristics :

If the speed and exciting current remain constant, the terminal voltage of the alternator changes with the load currents. **The plot between the terminal voltage and the load (or armature) current of an alternator is known as load characteristics.** An increase in the armature (or load) currents make the terminal voltage drops. This has been shown in Fig. 6. The drop in terminal voltage is attributed to many reasons but primarily because of the following:

- (a) Resistance and reactance of the armature (or stator) winding.
- (b) Armature reaction.

The resistance and reactance of the armature winding causes the drop in generated e.m.f. (voltage) ,whereas the armature reaction weakens the magnetic field and thereby decreases the generated e.m.f. (voltage).

The magnitude of the effect of armature reaction depends upon, the power factor of load i.e. angle of lag or lead of the stator (armature) current. In case of a unity power factor of load, each phase of alternator when connected to the load takes a current which is in phase with its generated voltage. But the magnetic field is strengthened, instead of weakening, if the load (or armature) current, is leading. In the above case, when the power factor is leading, the drop in voltage due to resistance and reactance of stator winding may be less than the increase in voltage due to armature reaction. Thus the terminal voltage on load may be more than that at no-load, if the angle of lead of the load (or armature) current is sufficient.



Fig. 6. Load Characteristic of alternator

#### (iii) Effect of variation of power factor on terminal voltage :

The load characteristics at different power factors with leading and lagging armature currents are shown in Fig. 7. If the load, current and excitation are kept constant, the terminal voltage falls on changing the power factor from leading to lagging one. This effect is because of the armature flux helping the main flux, in case p.f. is leading, hence generating more e.m,f. and the armature flux, opposing the main flux, in case the p.f. is lagging, hence generating less e.m.f., Therefore, the terminal voltage at lagging power factor decreases from that on leading p.f. because of decrease in generated e.m.f.



**Fig.** 7

#### Armature Leakage & Synchronous Reactances :

When the current pass through the stator conductors the flux is set up, and a portion of this flux does not cross the air gap but completes the path inside the stator as shown in Fig. 8. This flux is known as **leakage flux**, which sets up an e.m.f. in the stator winding .This e.m.f. leads the load current by 90° and proportional to the magnitude of load current. This e.m.f. is due to the leakage inductance of the armature winding. The magnitude of the leakage inductance in practical units, henrys, is given by the general equation :

L = (Flux in webers per amperes) \* (No. turns) henrys

And leakage reactance per phase,

### $X_l = \omega L = 2 \pi f L$ ohms/ph

 $X_l$  causes a voltage drop in alternator terminal voltage and this drop is equal to an e.m.f. set up by the leakage flux. Also, there is another source causes voltage drop, that is due to armature reaction which can be represented by a fictitious reactance  $X_a$ .

### $X_{l} + X_{a} = X_{s}$

Where **Xs** is the per phase synchronous reactance of armature winding.





### Performance Of A Round-Rotor Synchronous Generator :

At the outset we wish to point out that we will study the machine on a per phase basis, implying a balanced operation. Thus let us consider a round-rotor machine operating as a generator on no-load. Variation of the terminal voltage with the exciting current (field current) is shown in Fig. 5-I, and is known as the open-circuit characteristic of a synchronous generator. Let the open-circuit phase voltage be  $E_0$  for a certain field current  $I_f$ . Here  $E_0$  is the internal voltage of the generator. We assume that  $I_f$  is such that the machine is operating under unsaturated condition. Next we short-circuit the armature at the terminals, keeping the field current unchanged (at  $I_f$ ), and measure the armature phase current  $I_a$  In this case, the entire internal voltage  $E_0$  is dropped across the internal impedance of the machine. In mathematical terms,

 $E_o = I_a Z_s$ 

and  $\mathbf{Z}_s$  is known as the *synchronous impedance*, which is equal to

$$\mathbf{Z}_{\mathbf{s}} = \mathbf{R}_{\mathbf{a}} + \mathbf{j} \mathbf{X}_{\mathbf{S}}$$

If the generator operates at a terminal voltage  $V_t$ , while supplying a load corresponding to an armature current  $I_a$ , then

$$\mathbf{E}_{\mathbf{o}} = \mathbf{V}_{\mathrm{t}} + \mathbf{I}_{\mathrm{a}} \left( \mathbf{R}_{\mathrm{a}} + j \mathbf{X}_{\mathrm{S}} \right)$$

In an actual synchronous machine, except in very small ones, we almost always have  $X_S >> R_a$ , in which case  $Z_s \approx j X_S$ . Among the steady-state characteristics of a synchronous generator, its voltage regulation and power-angle characteristics are the most important ones.

We define the voltage regulation of a synchronous generator at a given load as

percent voltage regulation = 
$$(\mathbf{E}_{o} - \mathbf{V}_{t}) / \mathbf{V}_{t} * 100 \%$$

where  $V_t$  is the terminal voltage on load and  $E_o$  is the no-load terminal voltage.

The voltage regulation is dependent on the power factor of the load. the voltage regulation for a synchronous generator may even become negative. The angle between  $E_0$  and  $V_t$  is defined as the power angle,  $\delta$ . Notice that the power angle,  $\delta$ , is not the same as the power factor angle,  $\varphi$ . To justify this definition, we consider Fig. 9, from which we obtain

$$I_a X_S \cos \varphi = E_o \sin \delta \qquad \dots (i)$$



Fig. 9 Phasor diagram of round rotor generator

Now, from the approximate equivalent circuit (assuming that  $X_S >> R_a$  )as shown in Fig 10-a,

the power delivered by the generator = power developed,  $P_d = V_t I_a \cos \phi$ , which follows from Fig. 9 also. Hence, in conjunction with the (i) equation, we get

$$P_{d} = (E_{o} V_{t} / X_{S}) \sin \delta \quad \text{per phase} \quad \dots \text{(ii)}$$
$$P_{d} = 3(E_{o} V_{t} / X_{S}) \sin \delta \quad \text{for three phases}$$

Which shows that the internal power of the machine is porportional to  $\sin \delta$ , Equation (ii) is often said to represent the **power-angle characteristic** of a round rotor synchronous machine.

Fig. 10-b shows that for a negative  $\delta$ , the machine will operate as a motor.



Fig 10 : (a) an approximate equivalent circuit of synchronous machine (b) power-angle characteristics of a round-rotor synchronous machine

### Performance Of A Salient-Pole Synchronous Generator :

Because of saliency, the reactance measured at the terminals of a salient-rotor machine will vary as a function of the rotor position. This is not so in a round-rotor machine. Thus a simple definition of the synchronous reactance for a salient-rotor machine is not immediately forthcoming. To overcome this difficulty, we use the two-reaction theory proposed by "Andre Blondel". The theory proposes to resolve the given armature mmf's into two mutually perpendicular components, with one located along the axis of the rotor salient pole, known as the direct (or d )axis and with the other in quadrature and known as the quadrature (or q )axis. Correspondingly, we may define the d-axis and q-axis synchronous reactances,  $X_d$  and  $X_q$  for a salient-pole synchronous machine. Thus, for generator operation, we draw the phasor diagram of Fig. 11. Notice that  $I_a$  has been resolved into its d- and q-axis (fictitious) components,  $I_d$  and  $I_q$  With the help of this phasor diagram, we obtain

$$\begin{split} I_{d} &= I_{a} \sin \left( \delta + \phi \right) \\ I_{q} &= I_{a} \cos \left( \delta + \phi \right) \\ V_{t} \sin \delta &= I_{q} X_{q} = I_{a} X_{q} \cos \left( \delta + \phi \right) \\ \text{From these we get} \\ V_{t} \sin \delta &= I_{a} X_{q} \cos \delta \cos \phi - I_{a} X_{q} \sin \delta \sin \phi \\ \text{Or} \\ (V_{t} + I_{a} X_{q} \sin \phi) \sin \delta &= I_{a} X_{q} \cos \delta \cos \phi \\ \text{Dividing both sides by } \cos \delta \text{ and solving for } \tan \delta \text{ yields} \\ \tan \delta &= (I_{a} X_{q} \cos \phi) / (V_{t} + I_{a} X_{q} \sin \phi) \qquad \dots (\text{iii}) \\ \text{With } \delta \text{ known (in term of } \phi), \text{ the voltage regulation may be computed from} \\ E_{o} &= V_{t} \cos \delta + I_{d} X_{d} \end{split}$$



Percent regulation =  $(\mathbf{E}_{o} - \mathbf{V}_{t}) / \mathbf{V}_{t} * 100\%$ 

Fig 11 Phasor diagram of salient-pole generator

In fact, the phasor diagram depicts the complete performance characteristics of the machine.

Let us now use Fig. 11 to drive the power-angle characteristics of a salient-pole generator. If armature resistance is neglected,  $P_d = V_t I_a \cos \varphi$ , Now, from Fig. 11, the projection of  $I_a$  on  $V_t$  is

$$\begin{split} P_d / V_t &= I_a \cos \phi = I_q \cos \delta + I_d \sin \delta \\ \text{Solving} \\ I_q X_q &= V_t \sin \delta \qquad \text{and} \qquad I_d X_d = E_o - V_t \cos \delta \end{split}$$

For  $I_q$  and  $I_d$ , and substituting in (i), gives

 $\begin{aligned} P_{d} &= (E_{o} V_{t} / X_{d}) \sin \delta + (V_{t}^{2} / 2) [1 / X_{q} - 1 / X_{d}] \sin 2\delta & \text{per phase ...(iv)} \\ P_{d} &= 3 (E_{o} V_{t} / X_{d}) \sin \delta + 3 (V_{t}^{2} / 2) [1 / X_{q} - 1 / X_{d}] \sin 2\delta & \text{for three phases} \end{aligned}$ 

The equation can also be established for a salient-pole motor ( $\delta$ <0), the graph of above equation is given in Fig. 12. Observe that for  $X_d = X_q = X_s$ , reduces to the round-rotor equation.



Fig. 12 Power-angle characteristic of salient-pole machine

# **Parallel Operation of Synchronous Generators**

An electric power station often has several synchronous generators operating in parallel with each other. Some of the *advantages of parallel operation* are :

**1**. In the **absence** of the several machines, for maintenance or some other reason, the power station can function with the remaining units.

**2**. **Depending on the load**, generators may be brought on line, or taken off, and thus result in the most efficient and economical operation of the station.

3. For future expansion, units may be added on and operate in parallel.

In order that a synchronous generator may be connected in parallel with a system (or bus), the following *conditions must be fulfilled*:

*1*. The frequency of the incoming generator must be the same as the frequency of the power system to which the generator is to be connected.

2. The magnitude of the voltage of the incoming generator must be the same as the system terminal voltage.

3. With respect to an external circuit, the voltage of the incoming generator must be in the same phase as system voltage at the terminals.

4. In a three-phase system, the generator must have the same phase sequence as that of the bus.

The process of properly connecting a synchronous generator in parallel with a system is known as *synchronizing*. Tow generators can be synchronized either by using a synchroscope or lamps. Figure 1. shows a circuit diagram showing lamps as well as synchroscope. The potential transformers (PTs) are used to reduce the voltage for instrumentation. Let the generator  $G_1$  be already in operation with its switch  $S_{g1}$  closed. Other switches  $S_{g2}$ ,  $S_1$ , and  $S_2$  are all open.

After the generator  $G_2$  is started and brought up to approximately synchronous speed,  $S_2$  is closed. Subsequently, the lamps  $L_a$ ,  $L_b$ , and  $L_c$  begin to flicker at a frequency equal to the difference of the frequencies of  $G_1$  and  $G_2$ . The equality of the voltages of the two generators is ascertained by the voltmeter V, connected by the double-pole double-throw switch S. Now, if the voltages and frequencies of the two generators are the same, but there is a phase difference between the two voltages, the **lamps will glow steadily**. The speed of  $G_2$  is then slowly adjusted until the lamps remain permanently dark (because they are connected such that two voltages through them are in opposition). Next,  $S_{g2}$  is closed and  $S_2$  may be opened.



Fig 1. Synchronizing Two Generators

In the discussion above, it has been assumed that  $G_1$  and  $G_2$  both have the same phase rotation. On the other hand, let the phase sequence of  $G_1$  be **abc** counterclockwise and that of  $G_2$  be **a'b'c'** clockwise. At the synchronous speed of  $G_1$ , **a** and **a'** may be coincident. This will be indicated by a dark  $L_a$ , but  $L_b$  and  $L_c$  will have equal brightness, the phase rotation of  $G_2$  must be reversed. When  $G_2$  runs at a speed slightly less than the synchronous speed, with reverse phase sequence with respect to  $G_1$ , the lamps will be dark and bright in the cyclical order  $L_a$ ,  $L_b$  and  $L_c$ , the phase rotation of  $G_2$  must be reversed with increasing of its speed to synchronous speed. This process of testing the phase sequence is known as **phasing out**. A synchroscope is often used to synchronize two generators which have Previously been phased out. A synchroscope is an instrument having a rotating pointer, which indicates whether the incoming machine is slow or fast. One type of synchroscope is shown schematically in Fig. 2. It consists of a field coil,  $\mathbf{F}$ , connected to the main busbars through a large resistance  $\mathbf{R}_{\mathbf{f}}$  to ensure that the field current is almost in phase with the busbar voltage, V. The rotor consists of two windings  $\mathbf{R}$  and  $\mathbf{X}$ , in space quadrature, connected in parallel to each other and across the incoming generator.



Fig 2 A Synchroscope

The windings **R** and **X** are so designed that their respective currents are approximately in phase and 90° behind the terminal voltage,  $V_i$ , of the incoming generator. The rotor will align itself so that the axes of **R** and **F** are inclined at an angle equal to the phase displacement between V and V<sub>i</sub>. If there is a difference between the frequencies of V and V<sub>i</sub>, the pointer will rotate at a speed proportional to this difference. The direction of rotation of the pointer will determine if the incoming generator is running below or above synchronism. At synchronism, the pointer will remain stationary at the index. In present-day power stations, automatic synchronizers are used.

# **Circulating Current and Load Sharing**

At the time of synchronizing (that is, when  $S_2$  of Fig.1 is closed), if  $G_2$  is running at a speed slightly less than that of  $G_1$  the phase relationships of their terminal voltages with respect to the local circuit are as shown in Fig.3(a). The resultant voltage  $V_c$  acts in the local circuit to set up a circulating current  $I_c$ lagging  $V_c$  by a phase angle  $\varphi_c$  For simplification, if we assume the generators to be identical, then

$$\tan \varphi_c = R_a / X_s$$
$$I_c = V_c / 2Z_s$$

Where  $\mathbf{R}_a + \mathbf{J} \mathbf{X}_s$  = synchronous impedance ,  $\mathbf{R}_a$  = armature resistance ,  $\mathbf{X}_s$  = synchronous reactance.



Fig 3 Circulating Currents between Tow Generators

Notice from Fig. 3(a) that  $I_c$  has a component in phase with  $V_1$ , and thus acts as a load on  $G_1$  and tends to slow it down. The component of  $I_c$  in phase opposition to  $V_2$  aids  $G_2$  to operate as a motor and thereby  $G_2$  picks up speed. On the other hand, if  $G_2$  was running faster than  $G_1$  at the instant of synchronization, the phase relationships of the voltages and the circulating current become as shown in Fig. 3(b). Consequently,  $G_2$  will function as a generator and will tend to slow down; and while acting as a motor,  $G_1$  will pick up speed. Thus there is an inherent synchronizing action which aids the machines to stay in synchronism.

We now recall the power developed by a synchronous machine that  $V_t$  is the terminal voltage, which is the same as the system busbar voltage. The voltage E is the internal voltage of the generator and is determined by the field excitation. As we have discussed earlier, a change in the field excitation merely controls the power factor and the circulation current at which the synchronous machine operates. The power developed by the machine depends on the power angle  $\delta$ . For  $G_2$  to share the load, for a given  $V_t$  and E the power angle must be increased by increasing the prime-mover power. The load sharing between two synchronous generators is illustrated by the following examples.

# **Example 1:**

Two identical three-phase wye-connected synchronous generators share equally a load of 10 MW at 33 kV and 0.8 lagging power factor. The synchronous reactance of each machine is 6  $\Omega$  per phase and the armature resistance is negligible. If one of the machines has its field excitation adjusted to carry 125 A of lagging current, what is the current supplied by the second machine ? The prime mover inputs to both machines are equal.

### **SOLUTION**

The phasor diagram of current division is shown in Fig.4, where in  $I_1$ = 125 A. Because the machines are identical and the prime-mover inputs to both machines are equal, each machine supplies the same true power:



Fig 4

 $I_1 \cos \phi_1 = I_2 \cos \phi_2 = 0.5 \ I \cos \phi$ 

 $I = 10 * 10^6 / (\sqrt{3} * 33 * 10^3 * 0.8) = 218.7 A$ 

 $I_1 \cos \varphi_1 = I_2 \cos \varphi_2 = 0.5 * 218.7 * 0.8 = 87.5 A$ 

The reactive current of the first machine is therefore I<sub>1</sub> | sin  $\varphi_1$  | =  $\sqrt{(125^2 - 87.5^2)} = 89.3$  A

And since the total reactive current is I | sin  $\phi$  | = 218.7 \* 0.6 = 131.2 A

The reactive current of the second machine is  $I_2 | \sin \varphi_2 | = 131.2 - 89.3 = 41.9 \text{ A}$ 

Hence  $I_2 = \sqrt{(87.5^2 + 41.9^2)} = 97 \text{ A}$ 

### Example 2:

Consider the tow machines of example 1. if the power factor of the first machine is 0.9 lagging and the load is shared equally by the tow machines, what are the power factor and current of the second machine?

#### **SOLUTION**

Load:

Power = 10,000 KW, Apparent power =12,500 KVA, Reactive power = 7500 KVar

First machine:

Power = 5000 KW  $\phi_1 = \cos^{-1} 0.9 = -25.8^{\circ}$ Reactive power = 5000 tan  $\phi_1 = -2422$  KVar

Second machine:

Power = 5000 KW Reactive power = -7500 - (-2422) = -5078 KVar tan  $\varphi_2 = -5078 / 5000 = -1.02$  $\cos \varphi_2 = 0.7$  $I_2 = 5000 / (\sqrt{3} * 33 * 0.7) = 124.7$  A

# Determination of Synchronous Reactance from Open Circuit and Short Circut Tests

**i- Open Circuit Test:** The machine is run on no load and the induced e.m.f. per phase is measured corresponding to various values of field current and a curve between induced e.m.f. per phase,  $\mathbf{E}_{0}$  and field current,  $I_{f}$  is drawn which is known as open circuit characteristic (O.C.C.) and has been illustrated in Fig 5.



Fig 5 Open Circuit Characteristic

**ii- Short Circuit Test:** The armature winding is short-circuited through a low resistance ammeter. The speed is kept constant during this test and short-circuit current is measured corresponding to various values of field current. The field current or excitation is increased to give short-circuit current about twice the full load current. The short circuit characteristic (S.C.C.) is drawn by plotting a curve between short circuit current  $I_{sc}$  as ordinate and field current,  $I_f$  as abscissa, as shown in Fig. 5.

Consider OC the normal field current, then BC gives short circuit current,  $I_{sc}$  corresponding to this value of field current on the S.C.C. and AB gives the induced e.m.f. per phase on the O.C.C. for the same excitation. Since on short circuit for field current OC, the whole of the induced e.m.f. AC is utilized to create a short circuit current,  $I_{sc}$  given by BC.

Hence synchronous Impedance,

 $Z_s = AC$  (in volts) / BC (in amperes)

And synchronous reactance,

$$X_{s} = \sqrt{(Z_{s}^{2} - R_{a}^{2})}$$

Where  $\mathbf{R}_{\mathbf{a}}$  is the effective armature resistance per phase which can be measured directly by volt-meter and ammeter method or by using the wheat-stone bridge. For normal working conditions the armature resistance measured so is increased by 60% or so. This is being done to allow for skin effect and thus effective armature resistance  $\mathbf{R}_{\mathbf{a}}$  is obtained.

# Measurement of X<sub>d</sub> and X<sub>q</sub>

The d-axis synchronous reactance is determined from O.C. and S.C. tests, the q-axis synchronous reactance can be measured by many methods, one of these methods is the slip test method.

# Slip test method:

From this test, the value of  $X_d$  and  $X_q$  can be determined. The synchronous machine is driven by a separate prime-mover (or motor) at a speed slightly different from synchronous speed. The field windings are left open and positive sequence balanced voltages of reduced magnitude (around 25% of rated value) and of rated frequency are impressed across the armature terminals. Under these conditions, the relative velocity between the field poles and the rotating armature m.m f. waves is equal to the difference between synchronous speed and the rotor speed, i.e. the slip speed. A small A.C.voltage across the open field winding indicates that the field poles and rotating m.m.f. wave, are revolving in the same direction, and this is what is required in slip test. if field poles revolve in a direction opposite to the rotating m.m.f. wave, negative sequence reactance would be measured.

At one instant, when the peak of armature m.m.f. wave is in line with the field pole or direct axis, the reluctance offered by the small air gap is minimum as shown in fig. 6 (a). At this instant the impressed terminal voltage per phase divided by the corresponding armature current per phase, gives d-axis synchronous reactance  $X_{d}$ .

After one-quarter of slip cycle, the peak of armature m.m.f. wave acts on the interpolar or q-axis of the magnetic circuit, Fig. 6 (b). and the reluctance offered by long air-gap is maximum. At this instant, the ratio of armature terminal voltage per phase to the corresponding armature current per phase, gives q-axis synchronous reac<sup>t</sup> ance  $X_{q}$ . Oscillograms of armature current, terminal voltage and the e.m.f. induced in the open-circuited field winding are shown in Fig.7. A much larger slip than would be used in practice, has been shown in Fig.7, merely for the sake of clarity.



Fig. 6 pertaining to the physical concepts of (a)  $X_d$ , (b)  $X_q$ 



Fig. 7 Typical oscillograms in slip test

When the armature m.m.f. wave is along the direct axis, the armature flux passing through open field winding is maximum, therefore, the induced field e.m.f., i.e.  $d\phi_a/dt$  is zero. The d-axis can, therefore, be located on the oscillogram of Fig.7 where the induced field e.m.f. is zero. When armature m.m.f. wave is along q-axis, the armature flux linking the field winding is zero, therefore, the induced field e.m.f.  $d\phi_a/dt$  is maximum. Thus the q-axis can also be located on the oscillogram. If oscillograms can't be taken, then an ammeter and a voltmeter are used as shown in the connection diagram of Fig.8. The primemover (or d.c. motor) speed is adjusted till ammeter and voltmeter pointers

swing slowly between maximum and minimum positions. Under this condition, maximum and minimum readings of both ammeter and voltmeter are recorded in order to determined  $X_d$  and  $X_q$ .



Fig 8 Slip-test connection diagram for obtaining  $X_d$  and  $X_q$ 

Since the applied voltage is constant, the air-gap flux would be constant. When crest of the rotating m.m.f. wave is in line with the field-pole axis, Fig. 6 (a), minimum air-gap offers minimum reluctance, consequently the armature current, required for the establishment of constant air-gap flux, must be minimum. Constant applied voltage minus the minimum impedance voltage drop (armature current being minimum) in the leads and 3-phase variac gives maximum armature-terminal voltage. Thus the d-axis, synchronous reactance is given by

 $X_d = (Max. armature terminal voltage per phase) / (Min. armature current per phase)$ 

By a similar thought process

 $X_q = (Min. armature terminal voltage per phase) / (Max. armature current per phase) During slip test, it would be observed that swing of the ammeter pointer is very wide, whereas the voltmeter has only small swing because of the low impedance voltage drop in the leads and 3-phase variac. Since low armature-$ 

terminal voltages are used, values of reactances obtained are unsaturated values.

When performing this test, the slip should be made as small as possible, otherwise the currents induced in the amortisseur circuits would cause large errors in the measurement of  $X_d$  and  $X_q$  (lower value of reactances for larger slips). It is however quite difficult to maintain very small slips, as the reluctance torque due to saliency tends to bring the rotor into synchronism with the rotating armature m.m.f. wave. It is because of this reason that the slip test must be conducted at low values of armature terminal voltage so that reluctance torque due to saliency is low.

The advantages of oscillographic method over voltmeter-ammeter methods are

- i- elimination of the inertia effects of voltmeter and ammeter
- ii- the possibility of large slip-speed, which in turn allows higher armatureterminal voltages to be applied.

In practice, there may be error in reading the oscillograms. At the same time. voltmeter ammeter readings are not very reliable because of their inertia effect. In view of these shortcomings, slip test is conducted only to determine the ratio of  $X_d / X_q$ .

Now, using the value of  $X_d$  computed from o.c. and s.c. tests,  $X_q$  can be determined as follow :

 $X_q = X_d / X_q$  (from slip test) \*  $X_d$  (from O.C. and S.C. tests)

# **Voltage Regulation of Alternator**

From the previous discussion, it has been known that, the terminal voltage of an alternator changes from no load to full load.

The voltage regulation of an alternator is the difference between the no-load voltage and the full-load voltage exprsed in per cent of full load voltage.

*i.e.* percentage regulation =( $V_{no \ load} - V_{full \ load}$ ) /  $V_{full \ load} * 100\%$ 

$$= (E_o - V_t) / V_t * 100\%$$

Where  $E_o$  is no load terminal voltage,  $V_t$  is the full load rated terminal voltage

In case of leading power factor,  $E_0$  is less than  $V_t$  hence the regulation will be negative.

The regulation of an alternator is usually much higher than that of power transformers. This large regulation results from:

- the large amount of leakage reactance present in the alternator.
- the armature resistance.
- effect of armature reaction (this is the most predominant factor).

To make the generator voltage constant, automatic control on the field current (excitation) is necessary. A change in load will cause readjustment of excitation in sufficient magnitude so as to provide the constant-potential characteristic. The device used to control the field current (excitation) is called a voltage regulator. The regulator changes the alternator from a *variable-voltage* machine to a *constant-voltage* machine.

# 1. Determination of Voltage Regulation by Synchronous impedance method or E.M.F. method:

The magnitude of voltage regulation depends upon the load current and power factor of the load. The vector diagram for any load at any power factor can be drawn. This has been illustrated in Fig. 1.

Let I be the load current lagging behind the terminal voltage  $V_t$  by a phase angle  $\varphi$ .

- No load terminal voltage,  $E_0 = OF$
- Full load terminal voltage,  $V_t = OA$
- Armature resistance drop, I  $R_a = AC$ , this drop is always in phase with the load current I.
- Synchronous impedance drop,  $I X_s = CF$ , this drop is always perpendicular to the load current and also the resistance drop.



Fig. 1 Vector Diagram of Alternator

Now

•  $OF^2 = OD^2 + DF^2 = (OB + BD)^2 + (DC + CF)^2$ 

Or

•  $E_0^2 = (V_t \cos \phi + I R_a)^2 + (V_t \sin \phi + I X_s)^2$ 

Since

•  $AC = BD = I R_a$  And  $AB = CD = V_t \sin \phi$ Then

• 
$$E_0 = \sqrt{[(V_t \cos \phi + I R_a)^2 + (V_t \sin \phi + I X_s)^2]}$$

• percentage regulation =  $(E_0 - V_t) / V_t * 100\%$ 

For leading power factor of load, the phase angle  $\varphi$  will be taken as negative and the value  $\mathbf{E}_0$  will be even less than  $\mathbf{V}_t$  at large magnitude of leading power factor and the percentage regulation will be negative. For unity power factor taking phase angle  $\varphi$  as zero, the load current will be in phase, with the terminal voltage,  $\mathbf{V}_t$ . These cases have been illustrated in Fig. 2(a) and 2 (b).





(b) Leading power factor of load Fig 2

The regulation obtained by this method is always higher than the actual values and is therefore a pessimistic method. However, the results are more on the safe side and the method is theoretically accurate for non-salient pole

machines with distributed field windings when saturation is not considered. The value of synchronous reactance varies with the saturation. At low saturation its value is larger because of the effect of armature reaction is greater than that higher saturation. Under short circuit conditions, saturation is very low and the value of synchronous impedance (or reactance) measured is higher than that in actual working conditions.

## 2. Zero power factor method:

This is also called **the general method**, **Potier reactance (or triangle) method** of obtaining the voltage regulation.

In the e.m.f. method, the phasor diagram involving voltages is used, For the **z.p.f** method, the e.m.fs. are handled as voltages and the m.m.fs. as field ampere-turns or field amperes.

Zero-power-factor characteristic (**z.p.f c.**), in conjunction with **O.C.C.**, is useful in obtaining the armature leakage reactance  $X_l$  and armature reaction m.m.f.  $F_a$  For an alternator, **z.p.f c.** is obtained as follows

- The synchronous machine is run at rated synchronous speed by the primemover.
- A purer inductive load is connected across the armature terminals and field current is increased till full load armature current is flowing.
- The load is varied in steps and the field current at each step is adjusted to maintain full-load armature current. The plot of armature terminal voltage and field current recorded at each step, gives the zero -power- factor characteristic at full-load armature current.

From fig 3(a), it can be seen that the terminal voltage  $V_t$  and the air-gap voltage  $E_r$  are very nearly in phase and are, therefore, related by the simple algebraic equation

### $V_t = E_r - I X_l$

The resultant m.m.f.  $\mathbf{F}_r$  and the field m.m.f.  $\mathbf{F}_f$  are also very nearly in phase and are related by the simple algebraic equation

$$\mathbf{F}_{\mathbf{f}} = \mathbf{F}_{\mathbf{r}} + \mathbf{F}_{\mathbf{a}}$$

Assume that **O.C.C.** gives the exact relation between air-gap voltage  $E_r$  and the resultant m.m.f.  $F_r$  under load. Also, assume armature leakage reactance to be constant.

The O.C.C. and z.p.f c. are shown in Fig.3(b). For field excitation  $F_f$  or field current  $I_f$ , equal to OP the open-circuit voltage is PK. With the field excitation and speed remaining unchanged, the armature terminals are connected to a purely inductive load such that full load armature current flows. An examination of Fig. 3(a) and (b) reveals that under zero power. factor load, the net excitation  $F_r$  is OF which is less than OP (= $F_f$ )by  $F_a$  According to the resultant m.m.f. OF the air-gap voltage  $E_r$  is FC and if

**CB=IX**<sub>1</sub> is subtracted from  $E_r = FC$ , the terminal voltage  $FB = PA = V_t$  is obtained. Since z.p.f c. is a plot between the terminal voltage and field current  $I_f$  or  $F_f$ , which has not changed from its no-load value of OP, the point A lies on the z.p.f.c. The triangle ABC so obtained is called the *Potier triangle*, where CB=IX<sub>1</sub> and BA =  $F_a$ . Thus, from the Potier triangle, the armature leakage reactance  $X_1$  and armature reaction m.m.f.  $F_a$  can be determined.



Fig 3

# (a) General phasor diagram of Round-Rotor Alternator at zero power factor

#### (b) O.C.C., z.p.f c. And potier triangle

If the armature resistance is assumed zero and the armature current is kept constant, then the size of Potier triangle ABC remains constant and can be shifted parallel to itself with its corner C, remaining on the O.C.C. and its corner A, tracing the z.p.f.c. Thus the z.p.f.c. has the same shape as the O.C.C. and is shifted vertically downward by an amount equal to  $IX_1$  (i.e., ieakage reactance voltage drop) and horizontally to the right by an amount equal to the armature reaction m.m.f.  $F_a$  or the field current equivalent to armature reaction m.m.f.

For determining  $IX_l$  and  $F_a$  experimentally, it is not necessary to plot the entire z.p.f.c. Only two points A and F' shown in Fig 3(b) are sufficient. The point A(PA=rated voltage) is obtained by actually loading the alternator so that the rated armature current flows in the alternator. The other point F' on the z.p.f.c. corresponds to the zero terminal voltage and can, therefore, be obtained by performing short-circuit test. So here OF' is the field current required to circulate short-circuit current equal to the armature current (generally rated current) at which the point A is determined in the zero(near zero) power-factor test. Now draw a horizontal line **AD**, parallel and equal to **F'O**. Through point **D**, draw a straight line parallel to the air-gap line, intersecting the **O.C.C.** at **C**. Draw **CB** perpendicular to **AD**. Then **ABC** is the Potier triangle from which

$$BC = IX_1$$
$$AB = F_a$$

Since the armature current I at which the point A is obtained, is Known,  $X_1$  can be calculated.

Then determine the air-gap voltage  $E_r$  by the relation,

$$\mathbf{E}_{r} = \mathbf{V}_{t} + \mathbf{I} (\mathbf{R}_{a} + \mathbf{j} \mathbf{X}_{l})$$

According to the magnitude of  $\mathbf{E}_r$  obtain  $\mathbf{F}_r$  from **O.C.C.** and draw it leading  $\mathbf{E}_r$  by 90°, The armature reaction m.m.f.  $\mathbf{F}_a$  and armature leakage reactance  $\mathbf{X}_l$ , can be determined from the Potier triangle, as explained before. Now  $\mathbf{F}_a$  is drawn in phase with I as shown in Fig. 3(a) Then

$$\mathbf{F}_{\mathbf{f}} = \mathbf{F}_{\mathbf{r}} - \mathbf{F}_{\mathbf{a}}$$

is obtained and corresponding to  $F_f$ , excitation voltage  $E_f$  (or  $E_o$ ) is recorded from **O.C.C.** and the voltage regulation obtained.

Z.p.f. method requires O.C.C. and z.p.f.c., and gives quite accurate results.

# Example 1:

A three phase 6000 V Alternator gave the following open circuit characteristic at normal speed:

Field current in amper	14	18	23	30	43
Terminal Voltage in volt	4000	5000	6000	7000	8000

With the alternator short circuited and full load current folwing, the field current is 16 A. Neglecting the armature resistance and using synchronous impedance method, Determine the voltage regulation of the alternator supplying the full load of 200 KVA, at 0.8 power factor lagging.

### Solution

The open circuit characteristics have been drawn in fig 4, and corresponding to an excitation (field) current,  $I_f$  of 16 ampers, the open circuit voltage is 4700 V.

The phase O.C. voltage =  $4700 / \sqrt{3} = 2714 \text{ V}$ 

Full load current =  $(200 * 1000) / (\sqrt{3} * 6000 * 0.8) = 24$  A



Fig 4

Short curcuit current corresponding to field current of 16 ampers is 24 ampers Synchronuos impedance,

 $Z_s = O.C.$  voltage / S.C. current = 2714 / 24 = 113  $\Omega$ 

Now rated voltage of alternator per phase,

V= 6000 /  $\sqrt{3}$  = 3464 volts

And

$$Cos \phi = 0.8$$
  
Sin  $\phi = 0.6$   
I = 24 ampers

Load current,

$$\mathbf{E}_{o} = \sqrt{\left[\left(\mathbf{V}_{t} \cos \varphi + \mathbf{I} \mathbf{R}_{a}\right)^{2} + \left(\mathbf{V}_{t} \sin \varphi + \mathbf{I} \mathbf{X}_{s}\right)^{2}\right]} = \sqrt{\left[\left(3464 * 0.8 + 0\right)^{2} + \left(3464 * 0.6 + 24 * 113\right)^{2}\right]} = 5533 \text{ V}$$

Therefore,

Percentage regulation at full load =  $(E_o - V_t) / V_t * 100\%$ = (5533 - 3464) / 3464 \* 100= 59.7%

### Example 2:

A 220 V, 50 Hz, 6-pole star-connected alternator with, ohmic resistance of 0.06  $\Omega$ /ph, gave the following data for open-circuit, short-circuit and full-load zero-power-factor characteristics :

Find the percentage voltage regulation at full-load current of 40 amps at power-factor of 0.8 lag by (a) e.m f. method (b) z. p. f .

Field current, A	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.8	2.2	2.6	3.0
O.C. voltage, V	29	58.0	87.0	116	146	172	194	232	261.5	284	300
S.C. current, A	6.6	13.2	20.0	26.5	32.4	40.0	46.3	59	-	-	-
z.p.f. terminl voltage, v	-	-	-	-	-	0.0	29	88	140	177	208

#### Solution

Rated per phase voltage =  $220 / \sqrt{3} = 127 \text{ V}$ 

Per phase values for O.C.C. and z.p.f.c. are tabulated below and O.C.C., S.C.C. and z.p.f.c. are plotted in fig 5.

Field current, A	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.8	2.2	2.6	3.0
O.C. voltage, V	16.7	33.5	50.2	67	84.3	99.3	112	134	151	164	173
z.p.f. terminl voltage,v	_	-	-	-	-	0.0	16.73	50.8	80.8	102	120

A- E.M.F. Method:

 $\mathbf{Z}_{s} = \text{O.C. voltage / S.C. current}$ From above tables,  $\mathbf{Z}_{s} \approx 134 / 59 = 2.27 \Omega$ Here  $\mathbf{X}_{s} \approx \mathbf{Z}_{s} = 2.27 \Omega$ , since  $\mathbf{R}_{a}$  is quite small.

 $E_{o} = \sqrt{\left[ \left( V_{t} \cos \varphi + I R_{a} \right)^{2} + \left( V_{t} \sin \varphi + I X_{s} \right)^{2} \right]} \\ = \sqrt{\left[ (127 * 0.8 + 40 * 0.06)^{2} + (127 * 0.6 + 40 * 2.27)^{2} \right]} \\ = 196 \text{ volts}$ 

Percentage regulation =  $(E_o - V_t) / V_t * 100\%$ = (196 - 127) / 127 \* 100 = 54%

B- Zero power factor Method:

First of all, the Potier triangle **ABC** is drawn as described before, Point **A** corresponds to the rated voltage of 127 V on the **Z**. **p.f.c**. The line **AD** is drawn parallel and equal to  $\mathbf{F'O} = 1.2$  A. Then **DC** is drawn parallel to the air-gap line, meeting the **O.C.C.** at point **C**. Perpendicular **CB** on **AD**, lives **IX**<sub>1</sub> drop equal to 30 volts.

Armature leakage reactance  $X_l = 30 / 40 = 0.75 \Omega$ The air-gap voltage  $E_r$ ,  $E_r = V_t + I (R_a + j X_l) = 127(0.8 + j0.6) + 40*(0.06 + j0.75) = 148.6 \bot 45.6^\circ$   $Or = \sqrt{[(V_t \cos \varphi + I R_a)^2 + (V_t \sin \varphi + I X_l)^2]}$   $= \sqrt{[(127 * 0.8 + 40 * 0.06)^2 + (127 * 0.6 + 40 * 0.75)^2]}$ = 148.6 volts



Fig 5

Corresponding to  $\mathbf{E}_{\mathbf{r}} = 148.6$  V, the field current  $\mathbf{F}_{\mathbf{r}}$  from O.C.C. is 2.134 A, the armature m.m.f.  $\mathbf{F}_{\mathbf{a}}$ , from Potier triangle is  $\mathbf{AB} = 0.84$  A.

 $\begin{aligned} \mathbf{F_r} &= 2.134 \, \sqcup (45.6^\circ + 90^\circ) = 2.134 \, \sqcup 135.6^\circ \\ \mathbf{F_a} &= 0.84 \, \mathrm{A} \\ \mathbf{F_f} &= \mathbf{F_r} - \mathbf{F_a} = 2.134 \, \sqcup 135.6^\circ - 0.84 \\ &= 2.797 \, \sqcup 147.7^\circ \, \mathrm{A} \\ \end{aligned}$  For  $\mathbf{F_f} &= 2.797 \, \mathrm{A}$ , the excitation voltage from O.C.C. is  $\mathbf{E_o} = 169.0$  volts **Percentage regulation = (\mathbf{E\_o} - \mathbf{V\_t}) / \mathbf{V\_t} \* 100\% \\ &= (169 - 127) / 127 \* 100 = 33.1\% \end{aligned}** 

As already stated, **z.p.f.** method gives quite accurate results and here the voltage regulation with this method is 33.1%.

The voltage regulation by e.m.f. method is 54%, This value is much higher than the accurate value of 331% and in view of this, this method may be called pessimistic method.

# The Automatic Voltage Regulator

The method of operation in the power station is to maintain the terminal voltage of the alternator at the rated value and to adjust the excitation with change in load current accordingly. As the variations in load may be very violent both as regards magnitude and rapidity, it is clear that hand regulation of the excitation is impossible and that automatic means must be adopted. Now the flux per pole of a large turbo-alternator may amount to several webers, and therefore the self-induction of the field winding will be very high.

Consider, for example, a 15,000 kVA, 4 pole turbo-alternator having a flux per pole of 1.15 webers, a rotor current of 600 amperes, and a rotor winding of 66 turns per pole.

L(per pole)=(Flux per ampere) \* (No. of turns)

= (1.15 / 600) \* 66 = 0.127 henry

And Inductance, L = 4 \* 0.127 = 0.508 henry for the whole winding

Winding resistance,  $\mathbf{R}=0.2 \Omega$ 

The time constant = L / R = 0.508 / 0.2 = 2.54

The time constant  $\mathbf{L} / \mathbf{R}$  of the field circuit gives the value in seconds, and is the time taken for the flux to reach 0.6321 of its final value. With modern large two-pole, turbo-alternators the time constant is considerably greater. What is required in the power station that there shall be an almost instantaneous increase in excitation to the desired value. An increase in load calls for an increase in excitation. This increase is made much greater than is ultimately required, and as a result the flux builds up very rapidly. Before the flux can build up too far, the excitation is reduced again. This is known as the "overshooting-the-mark" principle. There are two types of quick acting regulator which work on this principle :

- (a) the vibrator type, in which a fixed resistance is rapidly cut in and out.
- (b) the rheostat type in which the resistance is variable.

# **Transistor-Cruntrolled or Transistor Type-Automatic Voltage Regulators**

The automatic voltage regulation is affected by matching a quantity proportional to the alternator voltage against a 'reference'. The difference between these two, called 'error' has to be rectified before it can be fed to the excitor, and this is affected by means of transistor amplifiers. This is a 'brushless' method of voltage control for an alternator. Here no slip rings, commutators and brush gear is required. These types of regulators are also called electronic voltage regulators The reference circuit is supplied by voltage feed back from the alternator, the general scheme of control has been illustrated in block diagram of fig 6.



Fig 6 Block diagram of the control system of a transistor-controlled alternator

# Three Phase Synchronous Motor

We know that the stator of a three-phase synchronous machine, carrying a three-phase winding currents produces a rotating magnetic field in the air gap of the machine. Referring to Fig. 1(a), we will have a rotating magnetic field in the air gap of the salient pole machine when its stator windings are fed from a three-phase source. Let the rotor (or field) winding be unexcited. The rotor will have a tendency to align with the rotating field at all times in order to present the path of least reluctance. Thus if the field is rotating, the rotor will tend to rotate with the field. From Fig 1(b) we see that a round rotor will not tend to follow the rotating magnetic field because the uniform air gap presents the same reluctance all around the air gap and the rotor does not have any preferred direction of alignment with the magnetic field. This torque, which we have in the machine of Fig 1(a), but not in the machine of Fig 1(b), is called the *reluctance torque*. It is present by virtue of the variation of the reluctance around the periphery of the machine.



### Fig. 1 (a) A Salient-rotor machine (b) A Round-rotor machine

Next, let the field winding [Fig. 1(a) or (b)] be fed by a dc source that produces the rotor magnetic field of definite polarities, and the rotor will tend to align with the stator field and will tend to rotate with the rotating magnetic field. We observe that for an excited rotor, a round rotor, or a salient rotor, both will tend to rotate with the rotating magnetic field, although the salient rotor will have an additional reluctance torque because of the saliency. In the past lecture we derive expressions for the electromagnetic torque in a synchronous machine attributable to field excitation and to saliency.

So far we have indicated the mechanism of torque production in a round-rotor and in a salient-rotor machine. To recapitulate, we might say that the stator rotating magnetic field has a tendency to "drag" the rotor along, as if a north pole on the stator "locks in" with a south pole of the rotor. However, if the rotor is at a standstill, the stator poles will tend to make the rotor rotate in one direction and then in the other as they rapidly rotate and sweep across the rotor poles. Therefore, a **synchronous motor is not self-starting**. In practice, as mentioned earlier, the rotor carries **damper bars** that act like the cage of an induction motor and thereby provide a starting torque. The mechanism of torque production by the damper bars is similar to the torque production in an induction motor. Once the rotor starts running and almost reaches the synchronous speed, it locks into position with the stator poles. The rotor pulls into step with the rotating magnetic field and runs at the synchronous speed; the damper bars go out of action. Any departure from the synchronous speed results in induced currents in the damper bars, which tend to restore the synchronous speed.

# Performance of a Round-rotor Synchronous Motor

Except for some precise calculations, we may neglect the armature resistance as compared to the synchronous reactance. Therefore, the steady-state per phase equivalent circuit of a synchronous machine simplifies to the one shown in fig 2(a). in which we show the terminal voltage  $V_t$ , the internal voltage  $E_o$ , and the armature current  $I_a$ , going "into" the machine or "out of" it, depending on the mode of operation "into" for motor and "out of" for generator. With the help of this circuit we will study some of the steady-state operating



Fig 2 : (a) an approximate equivalent circuit(b) power-angle characteristics of a round-rotor synchronous machine

characteristics of a synchronous motor. In Fig. 2(b) we show the power-angle characteristics as given by the power developed equation. Here positive power and positive  $\delta$  imply the generator operation, while a negative  $\delta$  corresponds to a motor operation. Because  $\delta$  is the angle between  $E_0$  and  $V_t$ ,  $E_0$  is ahead of  $V_t$  in a generator, whereas in a motor,  $V_t$  is ahead of  $E_0$ . The voltage-balance equation for a motor is, from Fig. 2(a),

 $\mathbf{V}_{t} = \mathbf{E}_{o} + j \mathbf{I}_{a} \mathbf{X}_{S}$ 

The per phase power developed is

 $P_{d} = (E_{o} V_{t} / X_{S}) \sin \delta$ 

If the motor operates at a constant power, then from above equations required that

$$I_a X_S \cos \varphi = E_o \sin \delta \qquad \dots (i)$$

We recall that  $E_0$  depends on the field current,  $I_f$ . Consider tow cases:

- (1) when  $I_f$  is adjusted so that  $E_0 < V_t$ , and the machine is underexcited.
- (2) when  $I_f$  is increased to a point that  $\mathbf{E}_o > \mathbf{V}_t$ , and the machine becomes overexcited.

The voltage-current relationships for the two cases are shown in Fig. 3(a). For  $E_o > V_t$  at constant power,  $\delta$  is greater than the  $\delta$  for  $E_o < V_t$ . Notice that an underexcited motor operates at a lagging power factor ( $I_a$  lagging  $V_t$ ), whereas an overexcited motor operates at a leading power factor.

In both cases the terminal voltage and the load on the motor are the same. Thus we observe that the operating power factor of the motor is controlled by varying the field excitation, hence altering  $E_0$ . This is a very important property of synchronous motors. The locus of the armature current at a constant load, as given by (i), for varying field current is also shown in Fig 3(a). From this we can obtain the variations of the armature current  $I_a$  with the field current,  $I_f$  (corresponding to  $E_0$ ), and this can be done for different loads, as shown in figures 3(b)and(c).



# Fig 3 (a) phasor diagram for motor operation (E<sub>o</sub>', I<sub>a</sub>', φ', δ') for underexcited operation, (E<sub>o</sub>", I<sub>a</sub>", φ", δ") for overexcited operation (b), (c) V-Curves of a synchronous motor

These curves are known as the *V*- *Curves* of the synchronous motor. One of the applications of a synchronous motor is in power factor correction, as demonstrated by the following examples. In addition to the V curves, we have also shown the curves for constant power factors. These curves are known as *compounding curves*. In the preceding paragraph, we have discussed the effect of change in the field current on the synchronous machine power factor. However, the load supplied by a synchronous machine cannot be varied by changing the power factor. Rather, the load on the machine is varied by instantaneously changing the speed (in case of a generator, by supplying additional power by the prime mover), and thus changing the power angle corresponding to the new load. In a synchronous motor, a load change results in a change in the power angle.

# EXAMPLE:

A three-phase wye-connected load takes 50 A of current at 0.707 lagging power factor at 220 V between the lines. A three-phase wye-connected, round-rotor synchronous motor, having a synchronous reactance of 1.27  $\Omega$  per phase. is connected in parallel with the load. The power developed by the motor is 33 kW at a power angle,  $\delta$ , of 30°. Neglecting the armature resistance, calculate (a) the reactive kilovolt-amperes (kVAR) of the motor and (b) the overall power factor of the motor and the load.

### Solution

The circuit and the phasor diagram on a per phase basis are shown in figures 4(a) and (b). From power developed equation, we have

$$P_d = 1/3 * 33,000 = 220/\sqrt{3} * E_o/1.27 * \sin 30^\circ$$

Which yield  $E_0 = 220$ 

And from phasor diagram,  $I_a X_S = 127$ , or  $I_a = 127/1.27 = 100$  A, and  $\varphi_a = 30^{\circ}$ The reactive kilovolt-amperes of the motor =  $\sqrt{3} * V_t I_a \sin \varphi_a$ 

$$=\sqrt{3*220/1000*100*\sin 30} = 19$$
 VAR

Notice that  $\phi_a$  is the power-factor angle of the motor,  $\phi_L$  is the power angle of the load,  $\phi$  is the overall power-factor angle, are shown in fig 4(b). The power angle  $\delta$ , is also shown in this phasor diagram, from which



Fig 4 (a) Circuit diagram (b) Phasor diagram

Or algebraically adding the real and reactive components of the currents, we obtain  $I_{real} = I_a \cos \phi_a + I_L \cos \phi_L$   $I_{reactive} = I_a \sin \phi_a - I_L \sin \phi_L$ The overall power factor angle,  $\phi$ , is thue given by  $\tan \phi = [I_a \sin \phi_a - I_L \sin \phi_L] / [I_a \cos \phi_a + I_L \cos \phi_L] = 0.122$ Or  $\phi = 7^\circ$  and  $\cos \phi = 0.992$  leading.

### **TORQUE – SPEED curve of a synchronous motor:**

In synchronous motor speed remains constant at Ns for all loads as in figure 5. At any other speed ( $\neq$  Ns) ,there is no motor –action , and therefore no torque will produced.



# Starting of Three Phase Synchronous Motor

As is clear from the construction of the motor, the stator of the synchronous motor is wound and the winding is connected to a.c. mains while the rotor of the motor is mostly of salient pole field construction except in special high speed two pole motors where it is non-salient pole field construction. The rotor is supplied from a d.c. source.

Consider Fig. 5, the stator for convenience of explanation is shown as having salient poles N' and S'. When the rotor is excited from d.c. source, there develop N and S poles on the rotor and this polarity is retained by the rotor throughout but the polarity of stator poles changes because it is connected to a.c. mains and the polarity alternates with the frequency of the a.c. supply.

First consider the rotor is stationary and in a position as shown in Fig. 6 (a). At this instant the similar poles of rotor and stator repel each other and the rotor tends to rotate in clockwise direction. But half a cycle later (i.e. 1/2f second) the polarity of the stator poles is reversed [as in Fig 6 (b)] but the polarity of rotor poles remains the same.



At this very instant unlike poles of stator and rotor attract each other and, therefore, the rotor tends to rotate back in the anti-clockwise direction. This shows that the torque acting on the rotor of the synchronous motor is not unidirectional but pulsating one. It is because of inertia of the rotor, it will not move in any direction. That is why the synchronous motor is not self-starting one.

Now let the rotor (which is yet unexcited) is speeded up to synchronous or near synchronous speed by some external arrangement and then it is excited through the d.c. source. The moment the rotor running at nearly synchronous speed is excited, it is magnetically locked into position with the stator poles which runs synchronously and that both in the same direction. Because of this interlocking between the rotor and stator poles, the synchronous motor has either to run synchronously or not at all.

# **Methods of Starting Synchronous Motors**

Since the three phase synchronous motor has no starting torque, so artificial means must be provided for starting it as below:

1. **External source.** If the d.c. field of a synchronous motor is supplied by a direct-connected exciter, this exciter can be used as a starting motor. This method is now rarely used.

2. **Induction-motor start.** If a squirrel cage winding is constructed in the pole faces of the synchronous motor (damper bars), it can be used to develop a starting torque (as well as provide damping) similar to that of the ordinary induction motor. As the motor reaches about 95 per cent of synchronous speed (it is operating as an induction motor with 5 percent slip) its field is excited and the motor pulls into step.

For such a set up, the problem of limiting the starting current without to low a value of starting torque is met in several ways:

- (a) **Auto-transformers** are used which are similar in design to those employed on induction motors. The stator is connected to the reduced voltage supply of the auto-transformer until synchronous or near synchronous speed is reached, and is then connected to the full voltage.
- (b) The voltage supplied to the stator can be reduced by using a **series reactors** in the supply lines. These reactors give a large voltage drop and low P.F. at starting.
- (c) Various types of rotor windings are used like double squirrel cage, or using special bar cross-sections (T bar, L bar, or simply deep, narrow bars) to give high skin effect to the squirrel cage and thus limit the starting current.
- (d) **Multiple winding.** This involves a special arrangement of stator coils so that two or more complete windings are paralleled for normal operation. The paralleled windings have normal values of reactance. If one winding is left open during starting process the resistance will double and the leakage reactance will be greater than the normal value and will result in a reduced current without the utilization of an autostarter. Switching arrangement is shown in Fig. 7.



(a) For starting, the short-circuited switch is open and only winding 1 is utilized

(b) The short circuiting switch closes the neutral of the second winding for parallel operation at normal load

### Fig 7 Arrangement of the double-winding synchronous motor

# **Synchronous Condenser**

When a three phase synchronous motor is used for power factor correction with no mechanical output (no load), it is known as a synchronous condenser. The term 'condenser' applied to device that it draws leading current as does a static condenser. There is a considerable increase in leading kilovolt- amperes available when the horsepower load of a synchronous motor is less than its rated load.

The synchronous condenser is especially designed so that practically all its rated kVA are available for p.f. correction. Because of the absence of shaft load, the mechanical design is modified from the ordinary motor standards. Because of the p.f. adjustment possible, the synchronous condenser is particularly applicable to transmission line control.

The ability to control the power factor of the synchronous motor by simply varying the excitation is of very great practical importance. The importance lies in the fact that the motor can be operated at a leading power factor when desired, and consequently if the remaining installation has a low power factor, the overall. power factor for the installation can be brought close to unity.

# Hunting and Damping of Synchronous Motor

When a synchronous motor is operating under steady-load conditions and an additional load is suddenly applied, the developed torque is less than that required by the load and the motor starts to slow down. A slight reduction in speed increases the phase angle of the generated voltage and permits more current to flow through the armature Some of the kinetic energy of the rotating parts is given out to the load during speed reduction . When the motor is slowing down, it cannot cease deceleration at exactly the correct torque angle corresponding to the increased load. It passes beyond this point, develops more than required torque, and increases in speed. This is followed by a reduction in speed and a repetition of the entire cycle. Such a periodic change in speed is called *hunting.* 

The mass of the rotating part and the 'spring effect' of the flux lines are the necessary elements which give the rotating field structure a natural oscillating period. If the load varies periodically with this same frequency or some multiple of it, the tendency is for the amplitude of these oscillations to increase cumulatively until the motor is thrown out of step, causing corresponding current and power pulsations. The mechanical stresses are likely to be severe, and the armature current greatly increases when the motor leaves synchronism.

A solid pole gives a **damping** action but has the disadvantage of excessive pole-face losses unless closed slots are used on the armature. The most satisfactory arrangement from the standpoint of simplicity seems to be laminated poles with the squirrel-cage 'amortisseur' or damping winding. The effectiveness of damper depends upon its resistance and to a lesser extent upon the length of the airgap. A low resistance damping winding produces the stronger effect, but if the synchronous motor is to be started by induction-motor action, using this squirrel cage, the winding resistance should be fairly high to produce good starting torque. Because of these opposing tendencies, a compromise usually must be reached in damping winding design.

### SHEET 5

### SOLVED EXAMPLES IN SYNCHRONOUS MACHINES

### Example 1 :

Calculate the percent voltage regulation for a three-phase wye-connected 2500 kVA 6600-V turboalternator operating at full-load and 0.8 power factor lagging. The per phase synchronous reactance and the armature resistance are 10.4  $\Omega$  and 0.071  $\Omega$ , respectively?

### Solution:

Clearly, we have  $X_s >> R_a$ , The phasor diagram for the lagging power factor neglecting the effect of  $R_a$  is shown in Fig. (a). The numerical values are as follows:  $V_t = 6600 / \sqrt{3} = 3810 \text{ V}$  $I_a = (2500 * 1000) / (\sqrt{3} * 6600) = 218.7 \text{ A}$  $E_o = 3810 + 218.7(0.8 - j 0.6) j10.4 = 5485 \bot 19.3^\circ$ Percentage regulation = (5485 - 3810) / 3810 \*100 = 44%

### Example 2:

Repeat Ex.1 calculations with 0.8 power factor leading as shown in Fig. (b) ? Solution:

 $E_o = 3810 + 218.7(0.8 + j 0.6) j10.4 = 3048 \bot 36.6^{\circ}$ Percentage regulation = (3048 - 3810)/3810 \* 100 = -20%



**<u>H.W.</u>**: Repeat Ex.1 calculations with unity power factor , and draw the phasor diagram for this case ?

### Example 3:

A 20-KVA, 220 V, 60 Hz, way-connected three phase salient-pole synchronous generated supplies rated load at 0.707 lagging power factor. The phase constants of the machine are  $R_a = 0.5 \Omega$  and  $X_d = 2 X_q = 4 \Omega$ . Calculate the voltage regulation at the specified load. ?

### Solution:

 $V_{t} = 220 / \sqrt{3} = 127 V$   $I_{a} = 20000 / (\sqrt{3} * 220) = 52.5 A$   $\varphi = \cos^{-1} 0.707 = 45^{\circ}$   $\tan \delta = (I_{a} X_{q} \cos \varphi) / (V_{t} + I_{a} X_{q} \sin \varphi)$  = 52.5 \* 2 \* 0.707 / (127 \* 52.5 \* 2 \* 0.707) = 0.37  $\delta = 20.6^{\circ}$   $I_{d} = 52.5 \sin (20.6 + 45) = 47.5 A$   $I_{d} X_{d} = 47.5 * 4 = 190 V$   $E_{o} = V_{t} \cos \delta + I_{d} X_{d}$   $= 127 \cos 20.6 + 190 = 308 V$ Percent regulation =  $(E_{o} - V_{t}) / V_{t} * 100\%$  = (308 - 127) / 127 \* 100% = 142 %

### Example 4:

The stator core of a 4-pole, 3-phase a.c. machine has 36 slots. It carries a short pitch 3-phase winding with coil span equal to 8 slots. Determine the distribution and coil pitch factors?

### <u>Solution:</u>

Number of slots per pole = 36 / 4 = 9

Angular displacement between slots =  $\beta = 180^{\circ} / 9 = 20^{\circ}$ 

Coil span = 20° \* 8 = 160°

Therefore, pitch factor =  $k_p = \cos(\theta/2)$  ,  $k_p = \cos 10^\circ = 0.985$ 

$$\theta = 180^{\circ} - 160^{\circ} = 20^{\circ}$$

Now, number of slots per pole per phase,  $\mathbf{m} = 9/3 = 3$ Distribution factor,  $\mathbf{k}_d = (\text{ Length of long chord }) / (\text{Sum of lengths of short chords })$   $= [\sin (\mathbf{m}\beta/2)] / [\mathbf{m} \sin (\beta/2)]$  $= \sin 30^\circ / (3 \sin 10^\circ) = 0.96$ 

#### Example 5:

3- phase, 50 Hz generator has 120 turns per phase. The flux per pole is 0.07 weber, assume sinusoidally distributed. Find (a) the e.m f. generated per phase. (b) e.m.f. between the line terminals with star connection. Assume full pitch winding and distribution factor equal to 0.96?

#### **Solution:**

(a) E.m.f. generated per phase =  $4 k_f k_p k_d f T \Phi_p$  volts

= 4 \* 1.11 \* 0.96 \* 1 \* 0.07 \* 50 \* 120

#### =1792 volts

(b) E.m.f. between line terminals =  $\sqrt{3}$  E.m.f. generated per phase =  $\sqrt{3} * 1792$ 

#### = 3100 volts

#### Example 6:

A three phase, 16-pole, star-connected alternator, has 192 stator slots with eight conductors per slot and the conductors of each phase are connected in series. The coil span is 150 electrical degrees. Determine the phase and line voltages if the machine runs at 375 r.p m. and the flux per pole is 64 mWb distributed sinusoidally over the pole?

#### Solution:

The flux is sinusoidally distributed over the pole, hence from factor,  $k_f = 1.11$ 

Pitch factor,  $k_p = \cos \left[ (180^\circ - 150^\circ) / 2 \right] = \cos 15^\circ = 0.966$ Distribution factor,  $k_d = \left[ \sin \left( \mathbf{m\beta} / 2 \right) \right] / \left[ \mathbf{m} \sin \left( \mathbf{\beta} / 2 \right) \right]$ 

 $\mathbf{m}$  = Number of slots per pole per phase = 192 / (16 \* 3) = 4

 $\beta = 180^{\circ}$  / (No. of slots/pole) =  $180^{\circ}$  / (192/16) =  $15^{\circ}$ 

 $k_d = [\sin (4 * 15^\circ / 2)] / [4 * \sin (15^\circ / 2)] = \sin 30^\circ / (4 * \sin 7.5^\circ) = 0.958$ Now total number of conductors per phase =Total number of slots per phase \* number of conductors per slot. = (192 / 3) \* 8 = 512

Number of turns per phase = T = 512 / 2 = 256

Frequency of generated e.m.f. = f = P N / 120

=[ Number of poles \* Synchronous Speed] / 120

=[16 \* 375] / 120 = 50 cycle per second

Flux per pole =  $\Phi_p$ =0.064 wb

Therefore, e.m.f. per phase =  $4 k_f k_p k_d f T \Phi_p$  volts = 4 \* 1.11 \* 0.966 \* 0.958 \* 0.064 \* 50 \* 256

= 3367 volts

Hence for star connected alternator, Line voltage =  $\sqrt{3} * 3367 = 5830$  volts

### Example 7:

A 2200 volt, star-connected, 50 cycle alternator has 12 poles. The stator has 108 slots each with 5 conductors per slot. Calculate the necessary flux per pole to give 2200 volts on no-load. The winding is concentric and the value of  $k_f$  can be taken as 1.11?

### **Solution**:

Slots per phase = 108 / 3 = 36

Therefore, conductors per phase = 36 \* 5 = 180

and number of turns per phase = 180 / 2 = 90

Slots per pole per phase = 36 / 12 = 12

Frequency, f = 50 cycle per second

 $k_f = 1.11$ 

 $k_p = 1.0$  for concentric winding

 $k_d = [\sin (\mathbf{m\beta}/2)] / [\mathbf{m} \sin (\mathbf{\beta}/2)]$ Where  $\mathbf{\beta} = 180^\circ$  / (No. Of slots/pole) =  $180^\circ$  / (108/12) =  $20^\circ$  $\mathbf{m}$  = Number of voltage vectors= Number of slots per pole per phase =  $108 / (12^*3) = 3$  $k_d = [\sin (3^*20^\circ/2)] / [3 \sin (20^\circ/2)] = [\sin 30^\circ] / [3 \sin 10^\circ] = 0.96$ Also e.m.f. per phase =  $\mathbf{E}_{ph} = 2200 / \sqrt{3}$  volts The e.m.f. equation for Alternator is  $\mathbf{E}_{ph} = 4 k_f k_p k_d f T \Phi_p$  volts  $\Phi_p$  = Flux per pole =  $\mathbf{E}_{ph} / [4 k_f k_p k_d f T] = (2200 / \sqrt{3}) / [4^*1.11^*10.96^*50^*90]$ = 0.066 wb

### Example 8:

Find the number of armature conductors in series per phase required for the armature of a 3 phase, 50 Hz, 10 pole alternator with 90 slots. The winding is to be star connected to give a line voltage of 11,000 volts. The flux per pole is 0.16 webers?

### Solution:

Number of slots per pole = 90 / 10 = 9

Therefore,  $\beta = 180^{\circ} / 9 = 20^{\circ}$ 

Slots per pole per phase,  $\mathbf{m} = 9/3 = 3$ 

Distribution factor,  $k_d = [\sin (\mathbf{m\beta}/2)] / [\mathbf{m} \sin (\mathbf{\beta}/2)]$ 

 $k_d = [\sin (3*20^{\circ}/2)] / [3 \sin (20^{\circ}/2)] = [\sin 30^{\circ}] / [3 \sin 10^{\circ}] = 0.96$ 

Since there is no mention about the type of winding, the full pitched winding is assumed.

Hence,  $k_p = 1.0$ 

Also,  $E_{ph} = 11000 / \sqrt{3} = 6352$  volts.

Now, 
$$\mathbf{E}_{ph} = 4 k_f k_p k_d f T \Phi_p$$
 volts  
 $T = [\mathbf{E}_{ph}] / [4 k_f k_p k_d f \Phi_p]$   
 $= [6352] / [4 * 1.11 * 0.96 * 1 * 0.16 * 50]$   
 $= 1863$ 

Number of Armature conductors in serves per phase = 1863 \* 2 = 3726

### Example 9 :

A six-pole, 3-phase, 60 cycle alternator has 12 slots per pole and 4 conductors per slot. The winding is 5/6 th pitched. There is 0.025 wb flux entering the armature from each north pole and the flux is sinusoidally distributed along the air gap. The armature coils are all connected in series. The winding is starconnected. Determine the open circuit e.m.f of the alternatorper phase?

### Solution:

Here, number of conductors connected in series per phase =12 \* 6 \* 4 / 3 = 96

Therefore, number of turns per phase, T = 96 / 2 = 48

Flux per pole ,  $\Phi_p = 0.025$  wb

Frequency, f = 50 cycle per second.

 $k_f = 1.11$ 

 $k_p = \cos \left[ 180^{\circ}(1 - 5/6) / 2 \right] = \cos 15^{\circ} = 0.966.$ 

Number of slots per pole per phase,  $\mathbf{m} = 12 / 3 = 4$ 

 $\beta = 180^{\circ} / 12 = 15^{\circ}$ 

Distribution factor,

 $k_d = [\sin (\mathbf{m}\beta/2)] / [\mathbf{m} \sin (\beta/2)]$ 

 $k_d = [\sin (4*15^{\circ}/2)] / [4 \sin (15^{\circ}/2)] = [\sin 30^{\circ}] / [4 \sin 7.5^{\circ}] = 0.958$ 

Hence induced e.m.f.,

 $E_{ph} = 4 k_f k_p k_d f T \Phi_p \text{ volts}$ = 4 \* 1.11 \* 0.966 \* 0.958 \* 0.025 \* 50 \* 48 = 295 volts.